Int. J. Heat Mass Transfer. Vol. 18, pp. 43–50. Pergamon Press 1975. Printed in Great Britain

THE THERMAL STRUCTURE OF FREE CONVECTION TURBULENCE FROM INCLINED ISOTHERMAL SURFACES AND ITS INFLUENCE ON HEAT TRANSFER

W. Z. BLACK
School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, U.S.A.

and

JACK K. NORRIS
U.S. Army 2nd Infantry Division, Seoul, Korea

(*Received* 9 *November* 1973 *and in revised form* 17 *May* 1974)

Abstract—A differential interferometer is used to provide flow visualization and measurement of the local heat-transfer coefficient for free convection from an inclined isothermal plate. The flow structure within the turbulent thermal boundary layer can be separated into a relatively constant thickness "thermal sublayer" and a core region that contains randomly fluctuating fluid typical of turbulent flow. The thermal sublayer is shown to contain "thermal waves" that traverse the surface of the heated plate and cause significant variations in the local heat-transfer coefficient. The frequency of occurance of the thermal waves increases in the transitional regime and is practically constant in the turbulent regime. The frequency of the thermal waves is decreased as the plate is inclined toward a horizontal position. Data for the time-average local Nusselt number for laminar, transitional and turbulent regimes are presented in addition to critical Rayleigh number values for the onset of transitional and turbulent flow.

NOMENCLATURE

f,	frequency of occurance of thermal waves;
h_x,	local convective heat-transfer coefficient;
Nu_x,	local Nusselt number;
Ra_x,	local Rayleigh number ($Gr_x Pr$);
t,	time;
x,	distance measured from leading edge of plate;
θ,	angle of plate inclination measured from vertical.

INTRODUCTION

THE FLOW structure within a turbulent boundary layer is quite complex and over the last decade or so much attention has been focused on the behavior of the fluid in the transitional and turbulent flow regimes. Attempts have been made to further understand the factors affecting flow stability. Wave theory [1] has been applied to determine conditions that will lead to turbulent flow.

Of the efforts to understand the structure of flow within the boundary layer, the behavior of the fluid near the solid surface has been carefully examined. It has been speculated [2] that the thin region near the wall plays a very important role in the structure of the entire turbulent boundary layer. The ejection of low speed fluid "bursts" from the region near the wall are thought to influence the transport of turbulent kinetic energy into the wake region. As a result, the behavior of the fluid in the vicinity of the surface is important to an understanding of the factors which affect the transport of mass and energy in turbulent situations.

Various experimental methods have been used to investigate the behavior of the fluid in the region adjacent to the solid surface. Two examples of the experimental techniques used are the injection of dye through slots in the wall and generation of hydrogen bubbles from a current carrying platinum wire [2, 3]. Both methods allow the measurement of the instantaneous velocity profile throughout the boundary layer. Other experimental studies have utilized a Mach–Zehnder interferometer [4] to observe the heated plume above a line source of heat in order to assess which frequencies of disturbances cause amplified oscillations in the plume and which decay as they are convected downstream in the plume. An electrochemical flow visualization technique is used in [5] to make visible flow instabilities which result in the onset of turbulent flow on the surface of an inclined plate.

The investigation reported in the present paper is experimental in nature and it uses a differential interferometer (frequently called a shearing interferometer) to provide a visual point-by-point record of the flow instabilities adjacent to the surface of a heated inclined plate. This study differs basically from previous efforts in that it attempts to correlate the existance of the flow instabilities in the regions close to the surface with fluctuations in the local convective heat-transfer coefficient. The results of the investigation are analyzed in an attempt to further understand the effects of the processes within the boundary layer and to determine how these processes relate to the heat transfer from the surface.

EXPERIMENTAL TECHNIQUE

A differential interferometer was utilized to provide visual patterns of the heated air near the test section and quantitative measurements of the local heat-transfer coefficient. Unlike the more frequently used Mach–Zehnder interferometer, the differential interferometer has not been widely applied to heat-transfer measurements. The differential interferometer produces a parallel fringe pattern in which the deflection of a fringe line is directly proportional to a temperature gradient existing in the test section. Infinite fringe patterns of a differential interferometer provide interference lines that are positions of constant temperature gradient. Therefore, rather than using a Mach–Zehnder interferometer to locate isotherms and approximating the temperature gradient by graphical means, the differential interferometer was used to provide a more direct measure of the convective heat-transfer coefficient.

For the purposes of this paper it is only important that the deflection of the parallel interference line is proportional to the local instantaneous convective heat-transfer coefficient. A complete description of the differential interferometer and a discussion of its capabilities and limitations when applied to heat-transfer measurements can be found in [6–8].

The test section selected for the study was an isothermal inclined flat plate. The plate was 1·27 cm thick aluminum with a surface area of 152 × 25·4 cm. The plate was heated by ten segmented electrical heaters attached to the rear surface of the plate. The energy input to each heater was varied until isothermal conditions existed throughout the plate. During each test the temperature of the plate was measured by thermocouples, fourteen of which were placed at various positions along the plate centerline and two of which were located off the center of the plate. All test data were collected for a plate temperature of 140°C with a variation in surface temperature no larger than 0·7°C.

The rear surface of the plate was insulated with fibreglass batten material and side barriers were placed along the entire plate length to prevent room air currents from disturbing the measurements. Since the interferometer provides a direct measure of the convective losses into the air, losses from the rear of the plate and radiative losses into an assumed transparent media do not affect the results. The plate was mounted in a carriage-rail system which allowed the plate to be inclined at a variable angle and moved through the viewing section of the interferometer so that measurements could be recorded over the entire 152 cm length. Measurements were recorded when the plate was inclined at 45°, 60°, 70° and 80° from the vertical. Reinforcing members of the interferometer prevented rotation of the plate into angles less than 45°. For this reason the flow differences between longitudinal vortices at angles greater than 15° and plane waves at angles less than 15° as reported in [5] could not be observed. Data were collected over the entire plate surface in the laminar, transitional and turbulent regions. The fluid for all tests was air.

The parallel and infinite fringe patterns were recorded with a 16 mm motion picture camera. The infinite fringe pattern provided a qualitative measure of the motion of the air within the boundary layer. Also the infinite fringe photos were used to determine the frequency of flow oscillations and the location for onset of transitional and turbulent flow regimes. The parallel fringe photographs were used to measure the local instantaneous convective heat-transfer coefficient. The local instantaneous values in the transitional and turbulent regions were integrated with time to obtain average Nusselt numbers. The flow visualization results and the heat-transfer measurements are presented in the following section.

RESULTS

Flow visualization

One very important benefit of using an interferometer in studies involving correlations between the flow structure of the fluid and the resulting effect on heat transfer is the fact that the fringe patterns produced by the interferometer permit visual observations of the flow of fluid surrounding the heated surface. In particular, the infinite fringe interferogram provides an instantaneous photograph of the thermal boundary layer and the type of flow which exists within the boundary layer. Regions of steady, laminar flow can easily be distinguished from the rapidly fluctuating flow that is characteristic of the turbulent regime.

The infinite fringe patterns were used to determine critical Rayleigh numbers which specify the beginning of the transition and turbulent regimes as a function of angle of plane inclination. The onset of transition was first marked in the infinite fringe photograph by a region of heated air that repeatedly rose from the surface and slowly disappeared into the outer region of the boundary layer. As the region of heated air moved into the boundary layer it was replaced by another region originating near the surface. The motion of the heated air at the onset of the transitional region formed a triangular shaped pattern with heated region anchored at the beginning portions of the transitional regime. As the air rose from the surface it appeared in the shape of a plane wave which was pivoted about the anchor point. The waves generated in this region formed the basis of the turbulent core fluid observed further up the surface of the plate.

While the onset of transitional flow was relatively easy to detect, the location of the fully turbulent regime was much more difficult to measure. The structure of the flow pattern in fringe photographs appeared very similar in the turbulent and transitional regimes. To provide a more exact method than the subjective test of observing the fluctuations in the fringe patterns, a method suggested by Lloyd *et al.* [9] was used to determine the critical Rayleigh number for turbulent flow. The method involves plotting the ratio of the local Nusselt number to the Rayleigh number raised to the one-third power as a function of the Rayleigh number. The dimensionless ratio $Nu_x/Ra_x^{1/3}$ should show no variation with Ra_x in the turbulent regime which is an indication that the heat-transfer coefficient is independent of distance from the leading edge of the plate. Since this behavior is indicative of turbulent flow, this criteria was used to determine the critical Rayleigh number for turbulent flow.

A summary of the critical Rayleigh numbers for the onset of transitional and fully turbulent flow as a function of surface inclination from the vertical is summarized in Table 1. Data for transitional critical Rayleigh numbers for plates inclined at 45° and 60° from [5] are included in Table 1 for comparison purposes. Critical Rayleigh numbers for turbulent flow and transition at high angles of inclination were unavailable for comparison.

Table 1. Critical Ra_x for onset of transitional and turbulent flow regimes

Angle θ (from vertical)	Transition Ra_x [5]	Observed transitional Ra_x	Observed turbulent Ra_x
45	$1{\cdot}7 \times 10^7$	$2{\cdot}4 \times 10^7$	$1{\cdot}5 \times 10^9$
60	$7{\cdot}7 \times 10^5$	$2{\cdot}0 \times 10^6$	$6{\cdot}2 \times 10^8$
70	—	$3{\cdot}0 \times 10^5$	$2{\cdot}8 \times 10^8$
80	—	$2{\cdot}0 \times 10^4$	$1{\cdot}0 \times 10^8$

The infinite fringe photographs were also used to determine the thickness of the thermal boundary layer surrounding the plate. The outer extent of the boundary layer grew quite rapidly and except for positions near the leading edge of the plate was always beyond the field of view of the interferometer. However, there was a "thermal sublayer" within the boundary layer that was relatively uniform in thickness regardless of distance from the leading edge of the plate. A composite photograph consisting of a number of individual still pictures has been assembled in Fig. 1 to illustrate this effect. The scale of the photo has been compressed in the direction along the surface of the plate so that the entire 152-cm plate length may be displayed in a single photo of reasonable size.

The fringe pattern is steady and consists of only the relatively constant thickness sublayer in the laminar region near the leading edge of the plate. In the transitional regime the exterior portion of the sublayer separates from the plate and grows rapidly in thickness. The thermal sublayer, however, remains practically constant in thickness throughout this region. The outer layer or core fluid shows a strongly turbulent behavior which has randomly fluctuating velocity components while the sublayer shows only minor fluctuations with time.

In the turbulent regime the thermal sublayer continued to be practically constant in thickness. The thickness of the sublayer for a plate inclined at 45° was within the range of 0·5–0·8 cm over the entire plate surface. The only significant variations in the thickness of the sublayer occurred when a "thermal wave" traversed the region separating the thermal sublayer and the core fluid. The disturbance is referred to here as a thermal wave because of the similarity between it and the appearance of wave motion in water. The severity of motion of the thermal wave became more significant and the degree of turbulence in the core fluid was much more apparent in the turbulent regime than in the transitional regime.

A single photograph of a thermal wave is shown in Fig. 2. The wave appears as a local thickening of the sublayer fluid. The extent to which the wave leaves the sublayer region increases as the wave moves further along the surface of the plate. Eventually the wave is forced from the sublayer region by the buoyancy forces which causes it to move into the core region. A portion of the thermal wave in Fig. 2 has moved into the core region and it appears as a distorted "smoke ring" which is being acted upon by the buoyancy force which attempt to move it vertically upward and the viscous forces of the bulk fluid which are attempting to move it in a direction more parallel to the surface of the plate. The light line in the photograph is a vertical reference and the two marks appearing in the lower portion of the photograph were set 2·54 cm apart for scaling purposes.

The appearance of a thermal wave is a result of an integrated effect across the heated surface in the direction of light propagation. The three-dimensional longitudinal waves observed by Lloyd and Sparrow [5] could not be observed due to the integrating effect although the observed thermal waves are felt to be an indication of heated air rising out of the longitudinal vortices.

procedure used to convert fringe deflections into heat-transfer coefficients is detailed in [8].

In the transitional and turbulent regimes the local heat-transfer coefficient showed significant variations with time. Figure 3 shows a typical plot of the local heat-transfer coefficient as a function of time at a location 51·5 cm from the leading edge of a plate inclined 70° from the vertical. The heat-transfer coefficient varies randomly with an unpredictable cyclic variation.

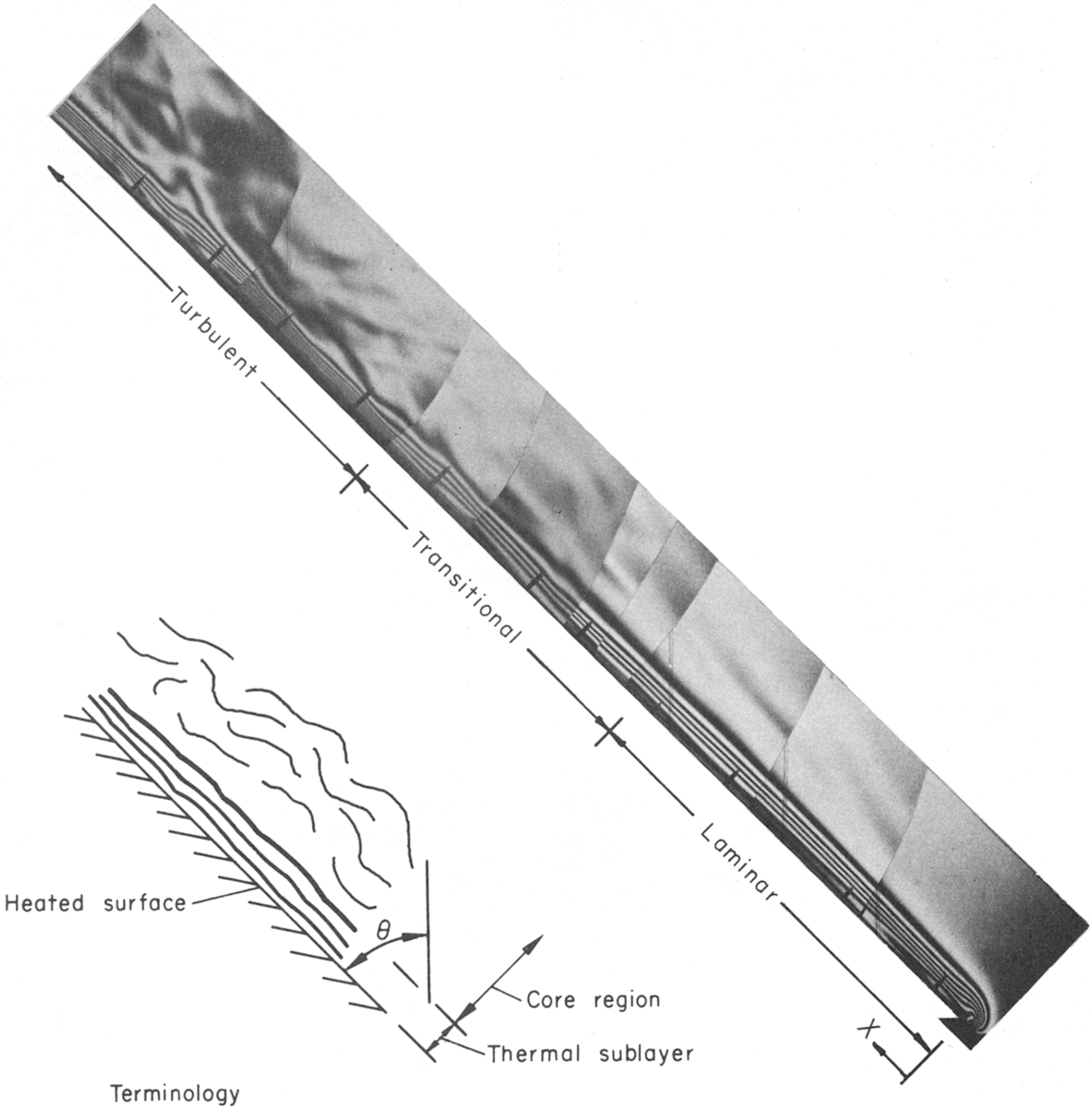

FIG. 1. Thermal boundary layer for 45° inclined plate showing thermal sublayer and core regimes.

Heat transfer

The parallel fringe pattern produced by the differential interferometer was used to measure the local convective heat-transfer coefficient in the laminar regime and the local instantaneous heat-transfer coefficient in the transitional and turbulent regimes. The test

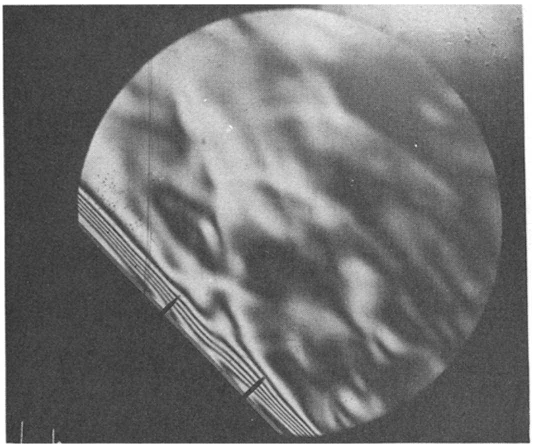

FIG. 2. Infinite fringe photograph showing thermal wave.

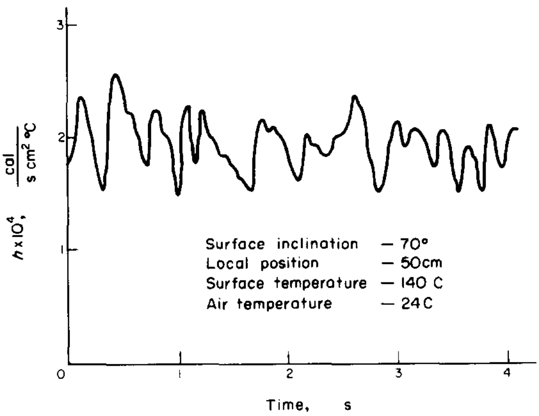

FIG. 3. Variation in local convective heat-transfer coefficient for turbulent flow.

The minimum values h_x were observed to occur at a time in which a thermal wave was moving past the local position on the plate. The frequency of wave occurance correlated well with the number of maximum values of h_x per unit time. In general the crest of the thermal wave coincides with a local region of decreased heat transfer. The wave crest causes a local thickening in the thermal boundary layer which results in a decrease in heat transfer in the vicinity of the wave. Immediately behind the wave as it moves up the plate is a region of increased heat transfer as is shown in Fig. 4. This curve is typical of the variation of heat transfer near the region of all thermal waves with the region of higher heat transfer being pulled along behind the region of low heat transfer by the motion of the wave. Typically the increase in h_x immediately behind the wave is relatively abrupt and the value decreases gradually further behind the wave until it reaches its minimum value near the location of the next thermal wave moving up the plate.

The frequency of thermal wave appearance at a local position on the plate surface was recorded for the four angles of inclination. A wave was counted at a given plate location each time a disturbance such as the one shown in Fig. 2 appeared in the infinite fringe photograph. Typical wave frequency data are shown in Fig. 5 for the first 100 cm of a plate inclined at 70° from the vertical. Regions of laminar, transitional and turbulent flow are also shown on the figure. No thermal waves were observed in the laminar region because the flow structure was steady. In the transitional regime the number of waves per unit time at a given location increased gradually until in the turbulent regime the frequency of wave occurance became constant. As a

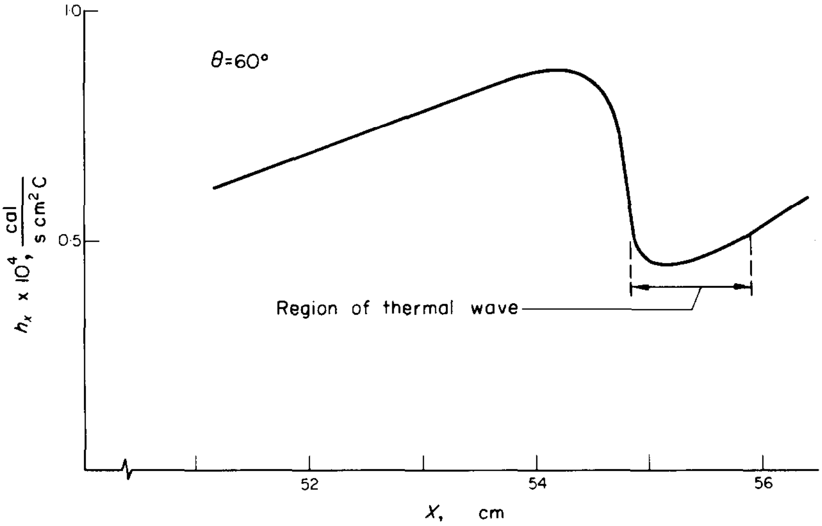

FIG. 4. Variation in local convective heat-transfer coefficient in the region of a thermal wave.

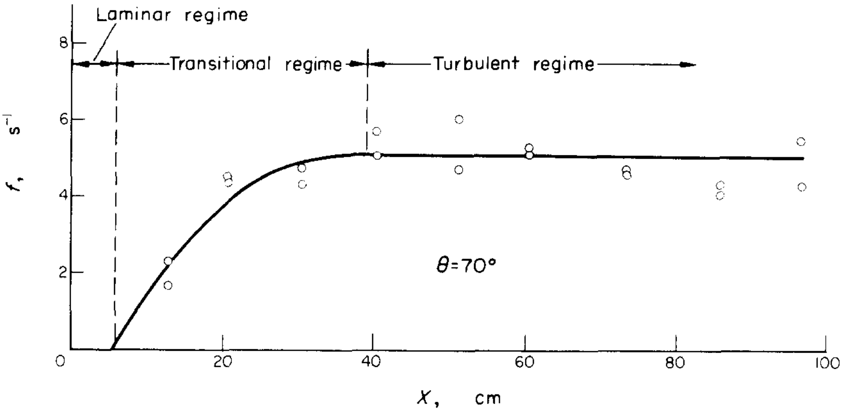

FIG. 5. Frequency of thermal waves in the three flow regimes.

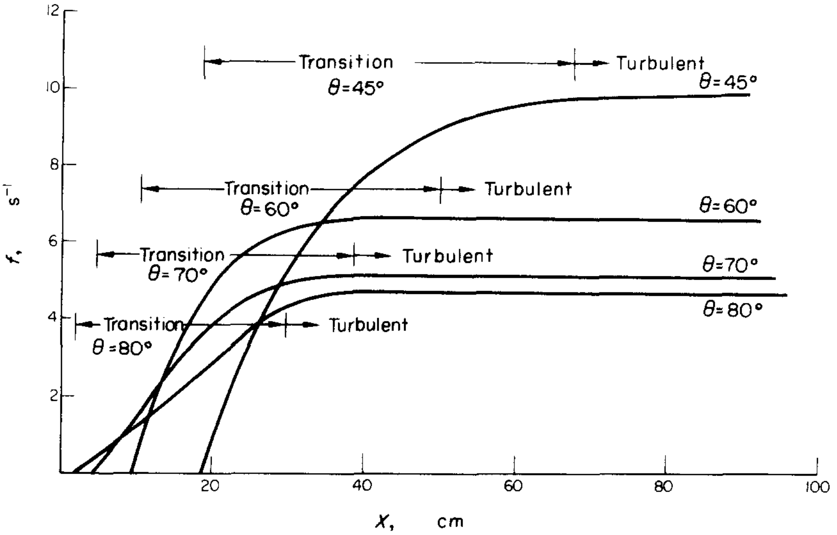

FIG. 6. Frequency of thermal waves for various plate inclinations.

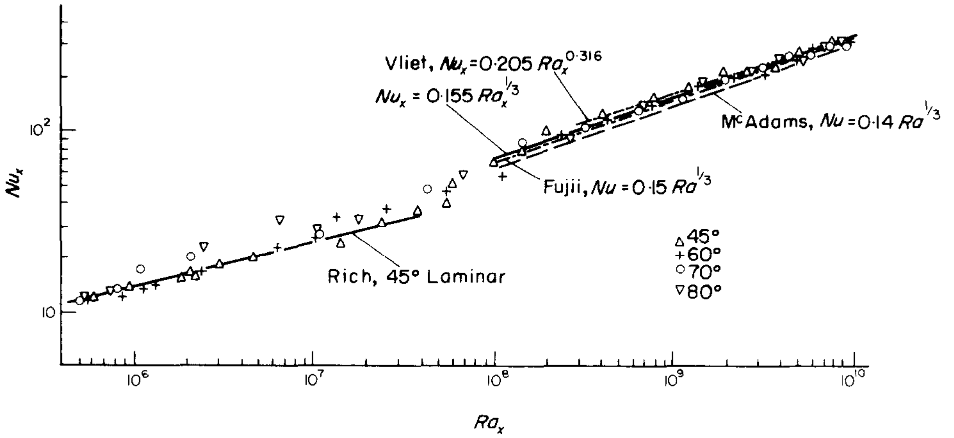

FIG. 7. Time-averaged Nusselt number for various plate inclinations.

result it appeared that more waves originated in the transitional regime than dissipated there and an equilibrium rate was reached in the turbulent regime where for each thermal wave which originated near the surface one left the sublayer and moved into the core fluid. Two sets of data are shown in Fig. 5 to give an indication of the scatter from one measurement to the next. Most of the scatter can be attributed to the appearance of numerous relatively weak thermal waves which may be counted as a wave in one test but not in the next.

The increase in wave frequency in the transitional regime and a constant frequency in the turbulent regime was typical behavior for all angles of inclination. A compilation of wave frequency for the four angles of plate inclination is made in Fig. 6. The frequency of thermal waves decreases with increasing inclination indicating that the most probable origin of the thermal waves is due to the shear forces caused by the faster core fluid rolling air over the slower moving fluid adjacent to the plate. This conclusion is basically the same as presented in [10] where the generation of free convection turbulence from a vertical plate is stated to be shear induced and the turbulent generation is stated to be governed by a strong outer generation region located away from the wall. Fewer waves occur as the plate is inclined toward the horizontal position since the shear forces are reduced and because those waves that are formed are more likely to be removed from the surface region by the buoyancy forces as the heated air attempts to move vertically upward.

The local instantaneous heat-transfer coefficient in the transitional and turbulent regimes was integrated with respect to time to obtain a time average local Nusselt number. Results appear in Fig. 7. The correlation for constant heat flux of Vliet [11], the correlation for horizontal isothermal plates of McAdams [12] and the data for inclined isothermal plates of Fujii [13] are included in the turbulent regime for comparison. The laminar data are also compared with the 45° correlation of Rich [14].

The data in the transitional regime showed some dependence on angle of inclination and they did correlate better when a Rayleigh number modified with the inclination or

$$(Ra_x)\cos\theta$$

was used instead of the unmodified Rayleigh number.

The data in the turbulent regime were insensitive to plate inclination and distance from the leading edge of the plate. The best fit curve in the turbulent regime was

$$Nu_x = 0{\cdot}155\,Ra_x^{1/3}$$

where the Rayleigh number does not include the dependency on θ. This correlation resulted in a standard deviation of all data points of $\pm 5{\cdot}9 \times 10^{-3}$ and a maximum deviation between the curve and all data points of $+5{\cdot}81$ and $-0{\cdot}9$ per cent. For all angles of inclination, the curve is about 10 per cent higher than the correlation for a horizontal plate, indicating that for turbulent flow over a plate inclined at 45°, the use of a horizontal plate correlation will result in a reasonably small error in the heat-transfer coefficient.

CONCLUSIONS

The differential interferometer provides a method of examining the free convection flow structure in the transitional and turbulent regimes. Furthermore it allows a simple method for measuring the correspondance between the fluctuations in the flow structure within the thermal boundary layer and their effect on the instantaneous heat-transfer coefficients. The occurance of thermal waves in the region near the plate has been shown to greatly influence the heat-transfer rate in the vicinity of the wave. The region near the wave is a location where the convective heat-transfer coefficient reaches a minimum value. The thermal waves are first generated within the transitional regime where they grow in number. In the turbulent regime the frequency of waves traversing the plate is relatively constant with the number of new waves generated adjacent to solid surface approximately equal to the number of waves which are removed from the thermal sublayer and move into the turbulent core fluid by the buoyancy forces.

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STRUCTURE THERMIQUE DE LA TURBULENCE EN CONVECTION NATURELLE POUR DES SURFACES ISOTHERMES ET SON INFLUENCE SUR LE TRANSFERT DE CHALEUR

Résumé—On utilise un interféromètre différentiel pour visualiser l'écoulement et mesurer le coefficient de transfert local, en convection naturelle sur une plaque isotherme inclinée. La structure de l'écoulement dans la couche limite thermique turbulente peut être séparée en une "sous-couche thermique" d'épaisseur à peu près constante et en un noyau qui contient les fluctuations désordonnées typiques de la turbulence. La sous-couche thermique contient des "ondes thermiques" qui traversent la surface de la plaque chauffée et qui provoquent des variations sensibles du coefficient de transfert local. La fréquence de ces ondes croît dans le régime de transition et est pratiquement constante dans le régime turbulent. Cette fréquence décroit quand la plaque est écartée de la position horizontale. On présente des résultats relatifs au nombre de Nusselt, moyen dans le temps, pour le régime laminaire, celui de transition et le régime turbulent, ainsi que les valeurs du nombre de Rayleigh critique correspondant à l'apparition de la transition et de l'écoulement turbulent.

DIE THERMISCHE STRUKTUR DER TURBULENZ BEI FREIER KONVEKTION AN GEGEIGTEN ISOTHERMEN OBERFLÄCHEN UND IHR EINFLUSS AUF DEN WÄRMEÜBERGANG

Zusammenfassung—Ein Differential-Interferometer wird für die Sichtbarmachung der Strömung und die Messung des örtlichen Wärmeübergangskoeffizienten bei freier Konvektion an einer geneigten isothermen Platte verwendet. Die Strömungsstruktur innerhalb der turbulenten thermischen Grenzschicht kann aufgeteilt werden in eine "thermische Unterschicht" von verhältnismaßig konstanter Stärke und in eine Kern-Region, die regellos fluktuierende Flüssigkeit—wie sie für turbulente Strömung typisch ist—enthält. Es wird gezeigt, daß die thermische Unterschicht "thermische Wellen" enthält, welche die Oberfläche der geheizten Platte überziehen und deutliche Unterschiede des örtlichen Wärmeübergangskoeffizienten verursachen. Die Frequenz, mit der die thermischen Wellen auftreten, nimmt im Bereich der Übergangsströmung zu und ist im turbulenten Strömungszustand praktisch konstant. Die Frequenz der thermischen Wellen nimmt in dem Maße ab, wie die Plattenneigung sich der horizontalen Position annähert.

Meßwerte der zeitlich gemittelten örtlichen Nusselt-Zahl bei laminarer, turbulenter und Übergangsströmung werden mitgeteilt, ebenso kritische Werte der Rayleigh-Zahl für den Beginn der Übergangs- und der turbulenten Strömung.

ТЕПЛОВАЯ СТРУКТУРА ТУРБУЛЕНТНОГО ПОГРАНИЧНОГО СЛОЯ ПРИ СВОБОДНОЙ КОНВЕКЦИИ НА НАКЛОННОЙ ИЗОТЕРМИЧЕСКОЙ ПОВЕРХНОСТИ И ЕЕ ВЛИЯНИЕ НА ТЕПЛООБМЕН

Аннотация — С помощью дифференциального интерферометра проведена визуализация потока и измерение коэффициента локального теплообмена от наклонной изотермической пластины. Структуру течения в тепловом турбулентном пограничном слое можно представить в виде «теплового подслоя» постоянной толщины и ядра, состоящего из беспорядочно пульсирующей жидкости, типичной для турбулентного течения. Показано, что тепловой подслой содержит «тепловые волны», которые пересекают поверхность нагретой пластины и вызывают значительные изменения коэффициента локального теплообмена. Частота возникновения тепловых волн возрастает в переходном режиме и практически постоянна в турбулентном режиме. Частота тепловых волн уменьшается с увеличением наклона пластины. Кроме значений критического числа Релея, при которых возникают переходное и турбулентное течения, в работе приводятся средние по времени локальные числа Нуссельта для ламинарного, переходного и турбулентного режимов.